Learning strategic interactions from individual actions

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MIT Connection Science

mit media lab

An increasingly connected society

- “six degrees of separation”

The Milgram experiment [1964]

An increasingly connected society

- "six degrees of separation"
- "three degrees of influence"

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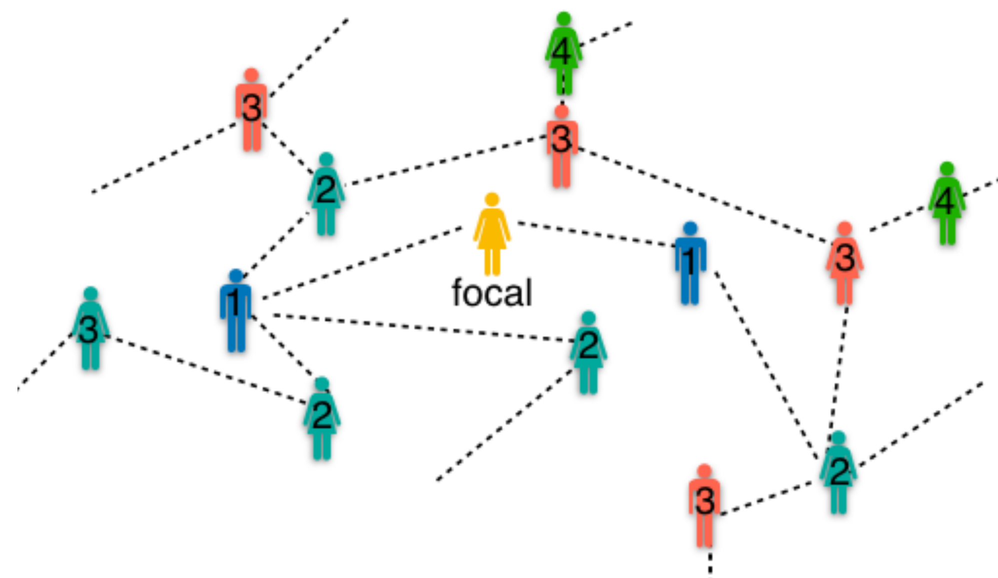

Three degree of influence
[Christakis and Fowland 2008]

An increasingly connected society

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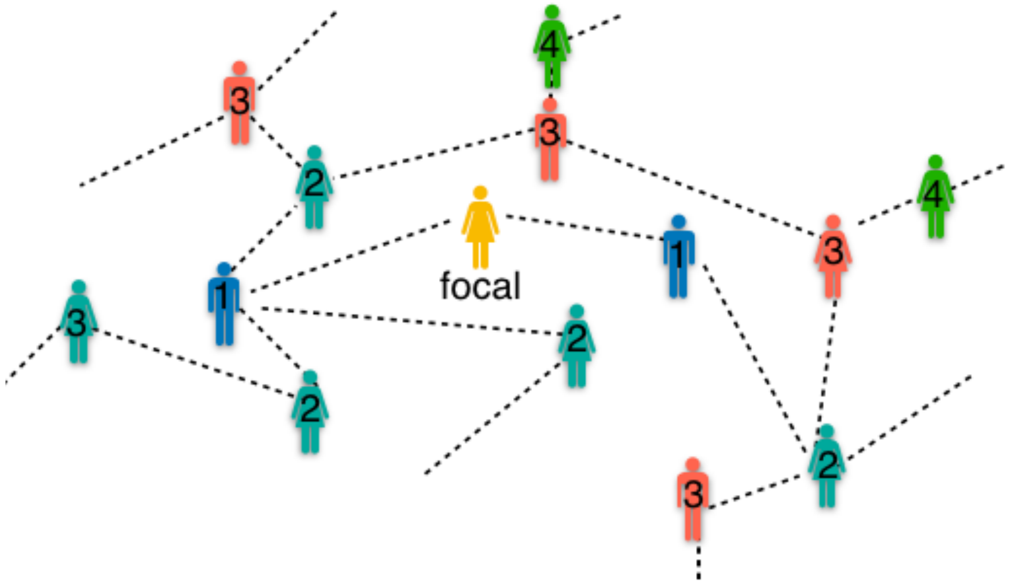

Three degree of influence
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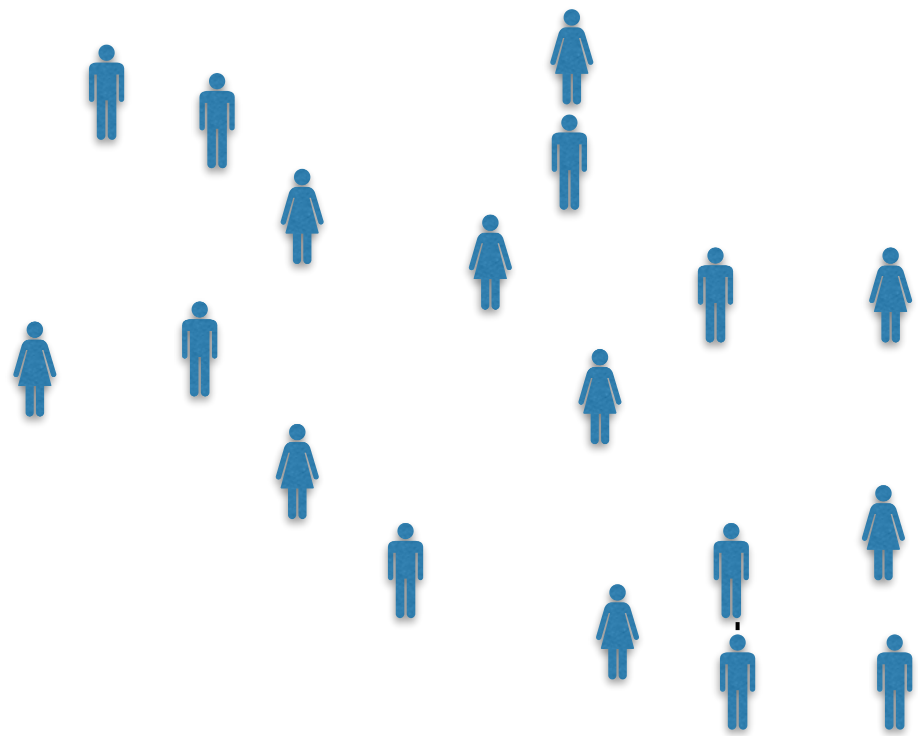
What does this mean for stakeholders & policy-makers?

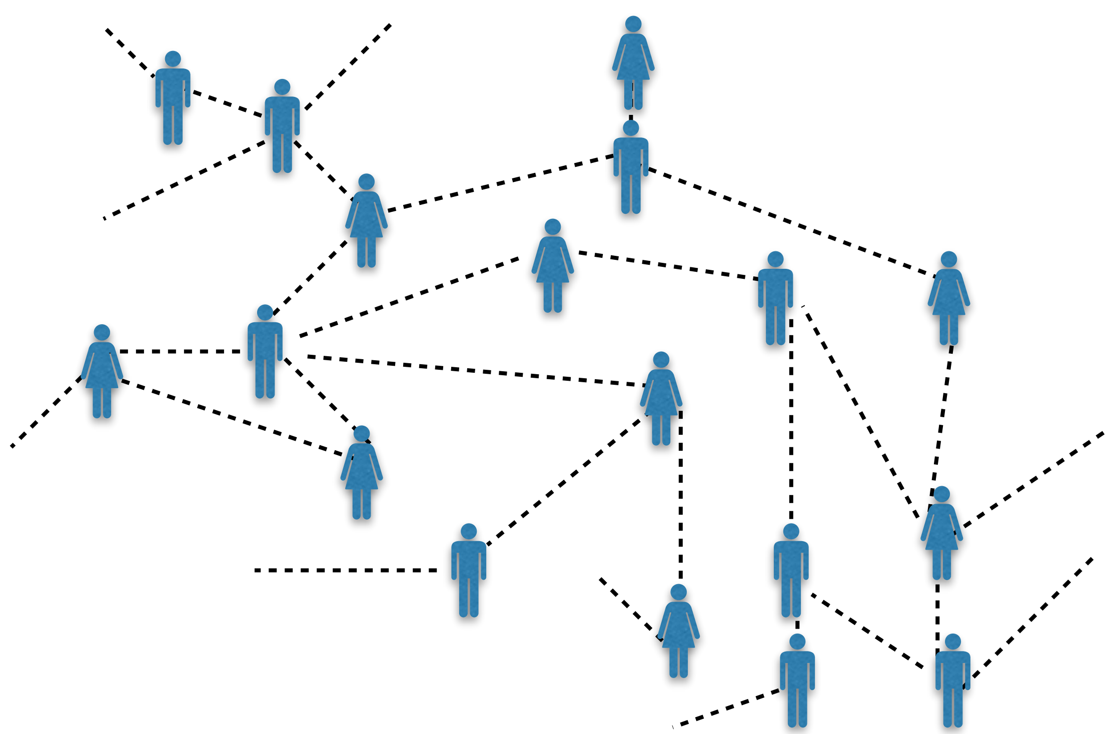
Need for networks in practice

- Vaccination: how to prioritize?
- Marketing campaign: whom to target first?



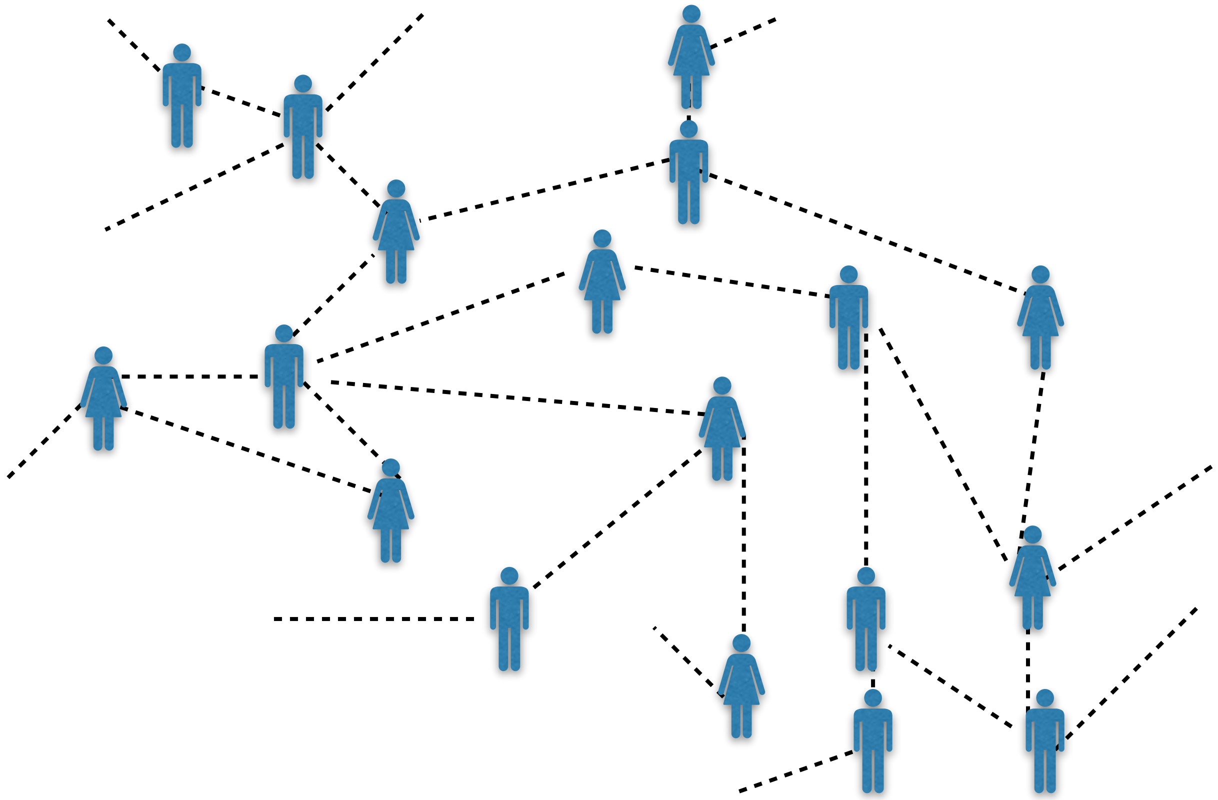
Need for networks in practice

- Vaccination: how to prioritize?
- Marketing campaign: whom to target first?
- Network interventions, for example:
  - inoculating whoever has most physical contacts
  - giving sample to whoever is most well-connected



Need for inferring networks

- Network interventions: the process of using social network data to accelerate behavior change or improve organizational performance [Valente 2012]



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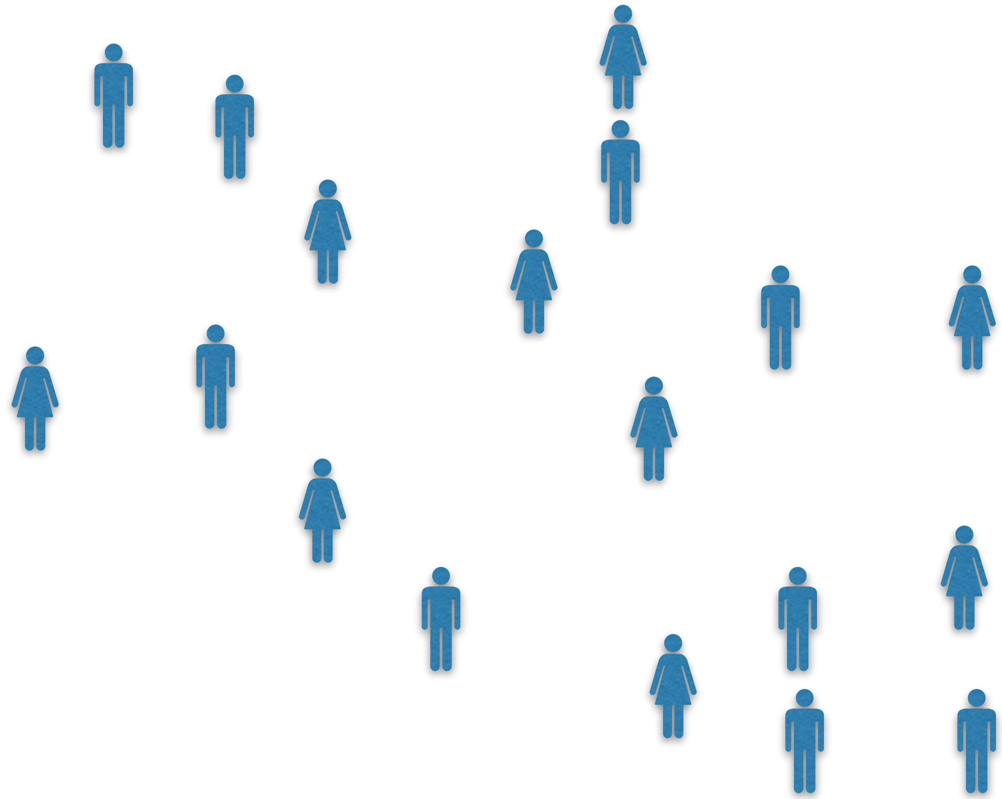
  - costly to collect
  - fuzzily defined and noisily measured
  - privacy concerns
  - dynamic



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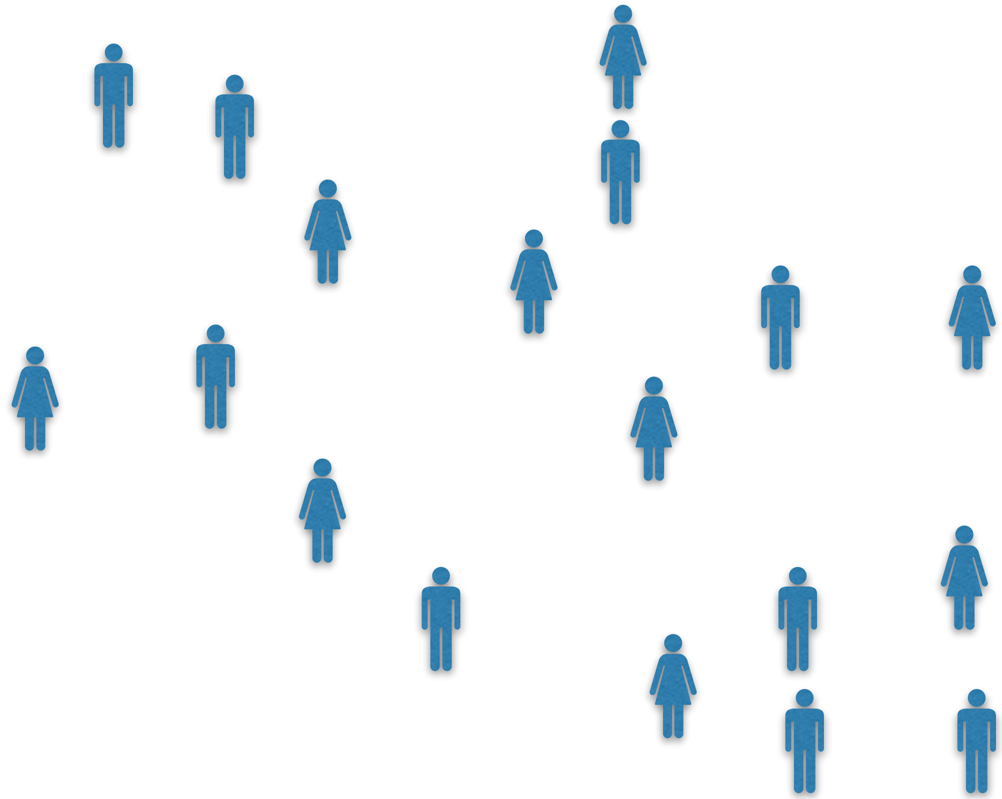

Infer the underlying relationships/ interaction network from the observed actions

Why can we infer relationships from decisions?

- People influence the decision-making of one another

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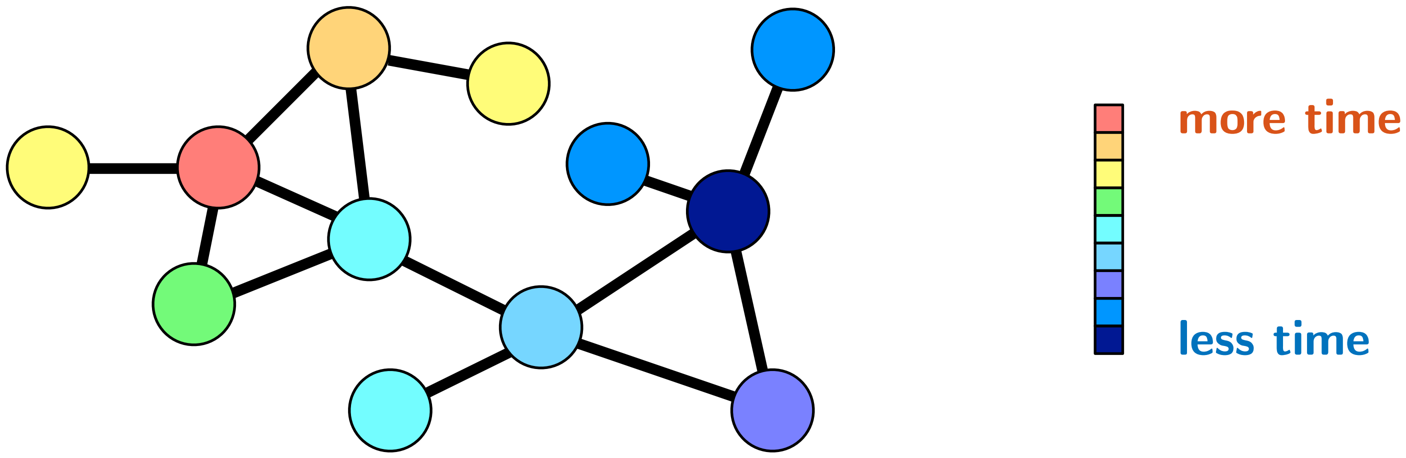

Complement

making choices how much time to spend on a social app

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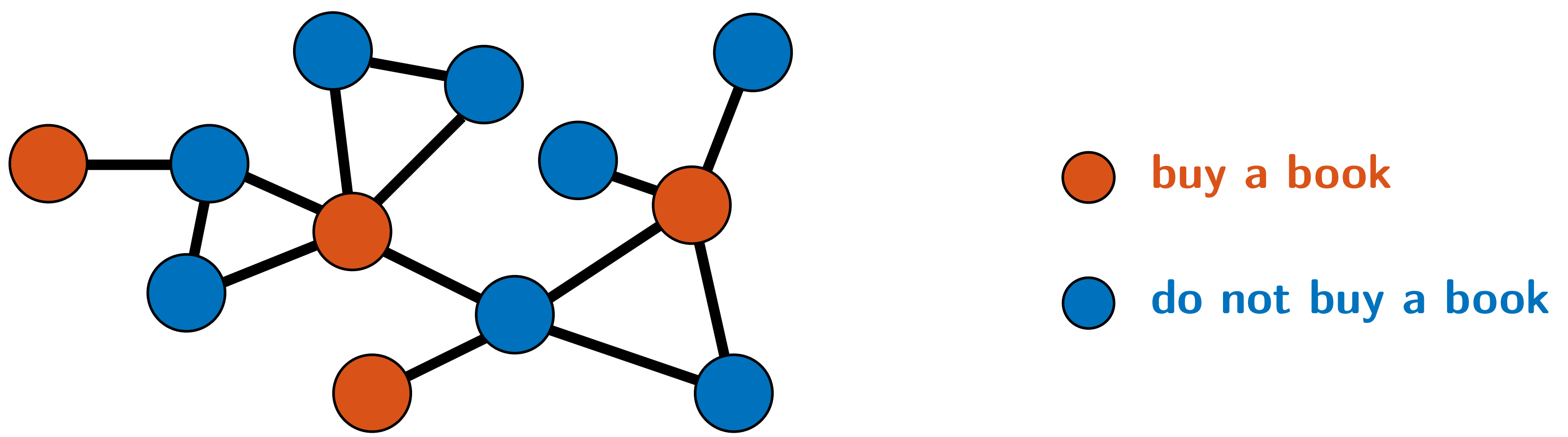
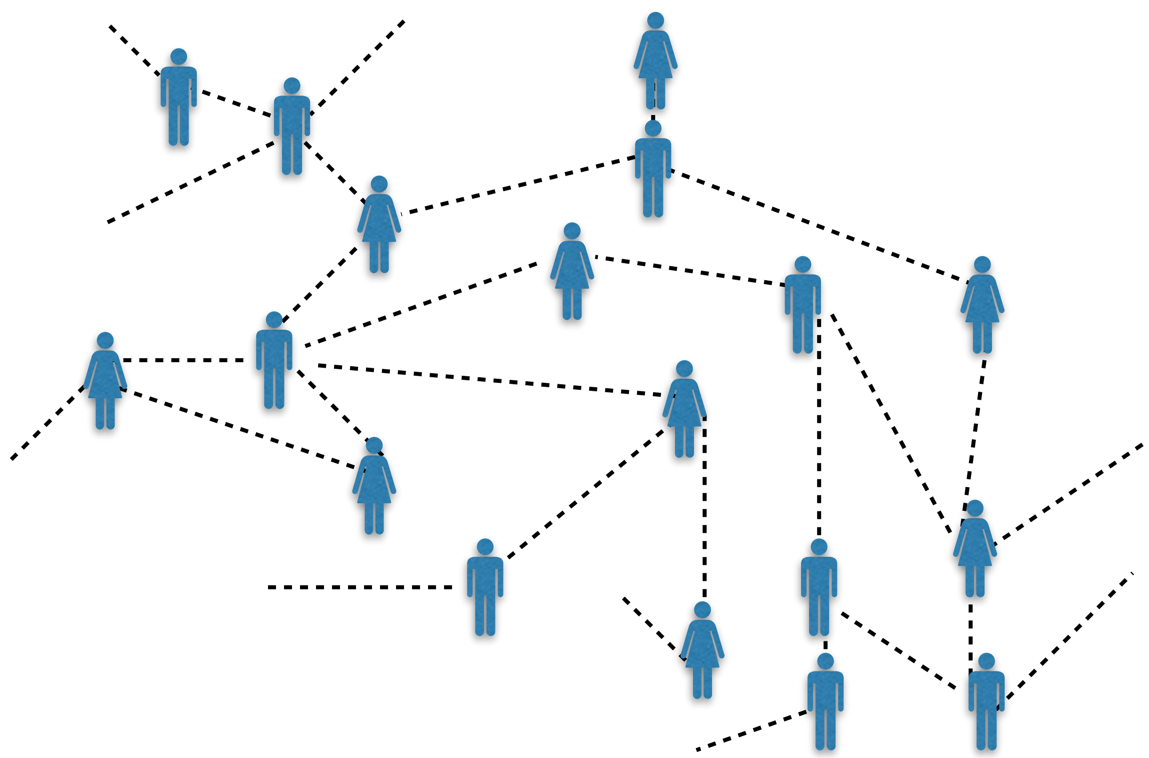


Substitutive

making choices on buying a book

Games on networks

- Such interactions can be modeled as games on networks
  - players, actions, payoff function, interaction network



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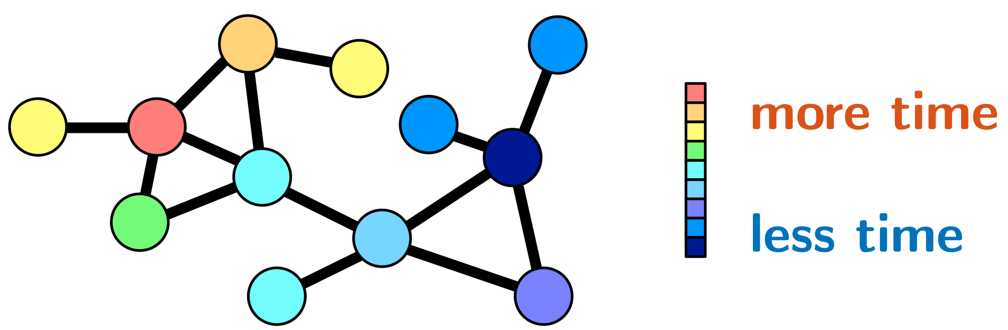
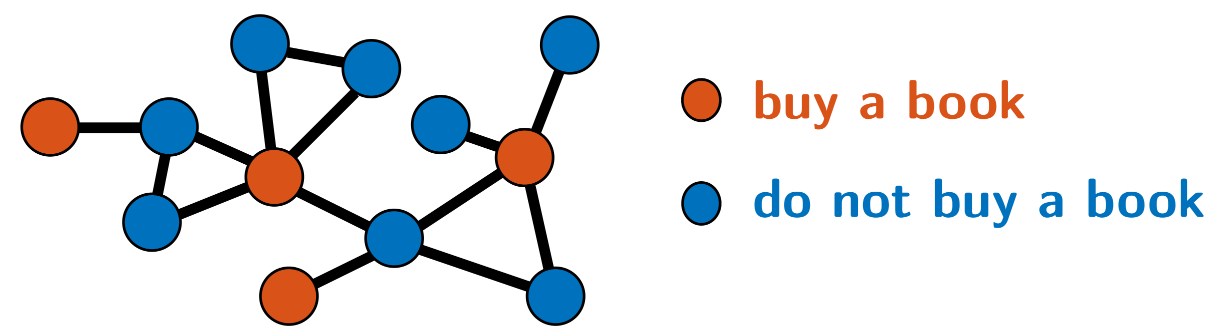
    - given player's relative payoff to taking an action (higher action) is **increasing** or **decreasing** in neighbors who take the action (higher action)





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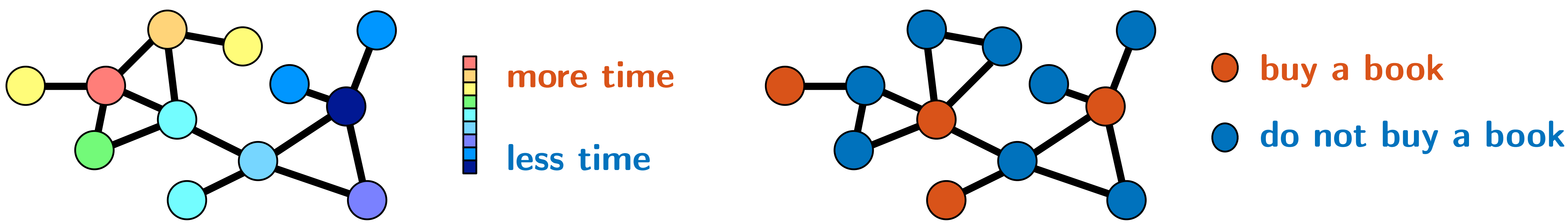

- Observe actions from strategic interactions:
  - **learning** interaction **network**
  - **learning marginal** benefits

Outline

- Setting
  - network games of linear-quadratic payoff
- Framework for learning network games
  - independent marginal benefits
  - homophilous marginal benefits
- Experimental results
  - synthetic data
  - real-world data
- Applications
- Discussions

Network games of linear-quadratic payoffs

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- Examples

	action (**a**)	payoff (**u**)
education	effort	achievement
collaboration	joint R&D activities	firm profit

Games with linear-quadratic payoffs

- Pure-strategy Nash equilibrium (PSNE)

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- Assumption: spectral radius of $\beta \mathbf{G}$ is smaller than 1
  - guarantees matrix inversion
  - ensures uniqueness and stability of equilibrium action [Ballester 2006]

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- Properties
  - equilibrium related to Katz-Bonacich centrality
  - payoff dependency spreads indirectly through network

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quadratic program jointly convex in G and B

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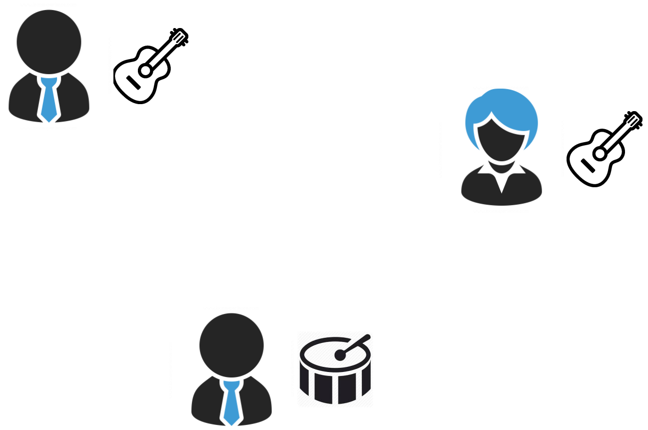

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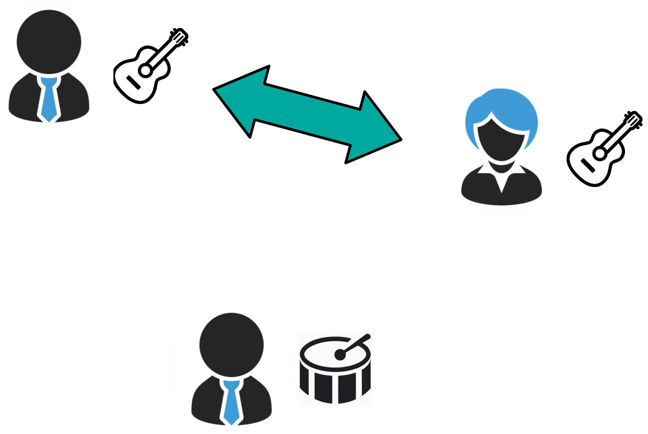

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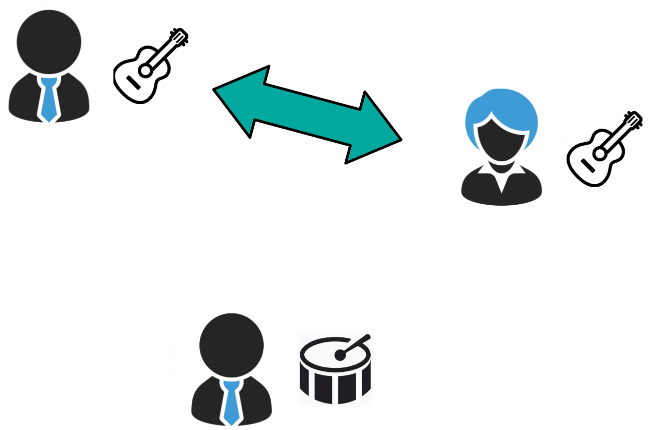

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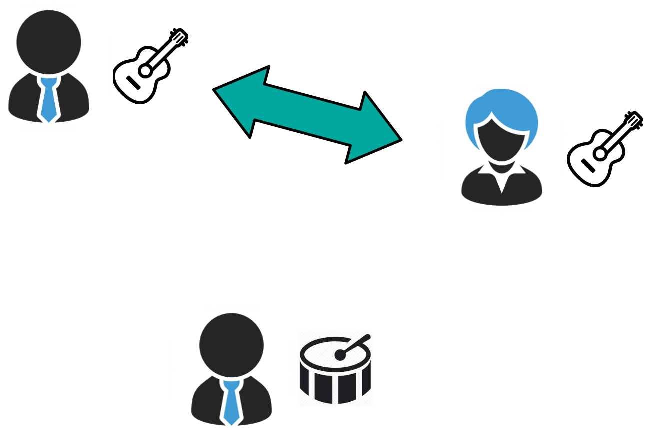

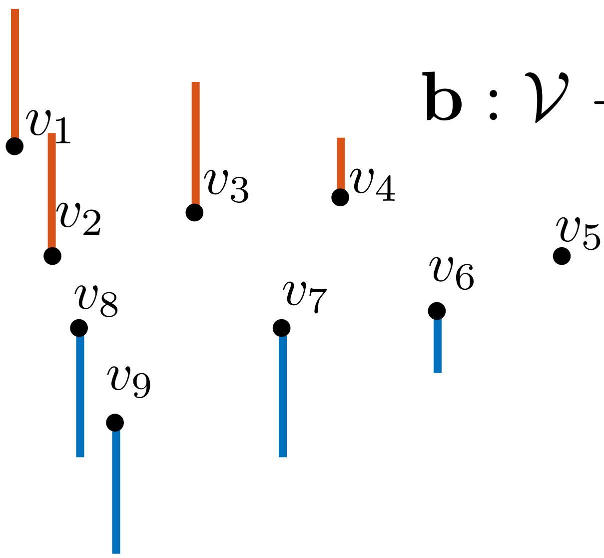

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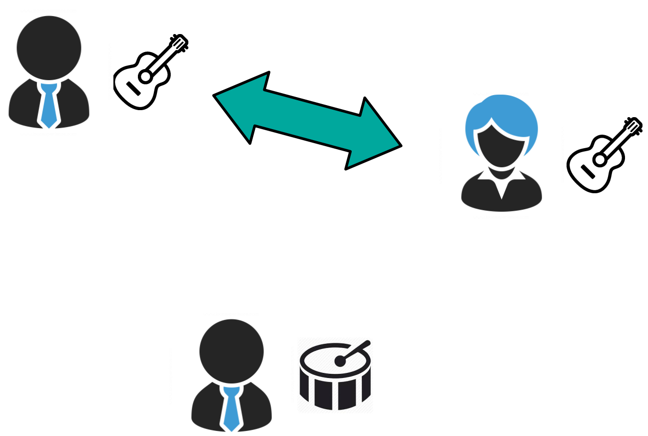

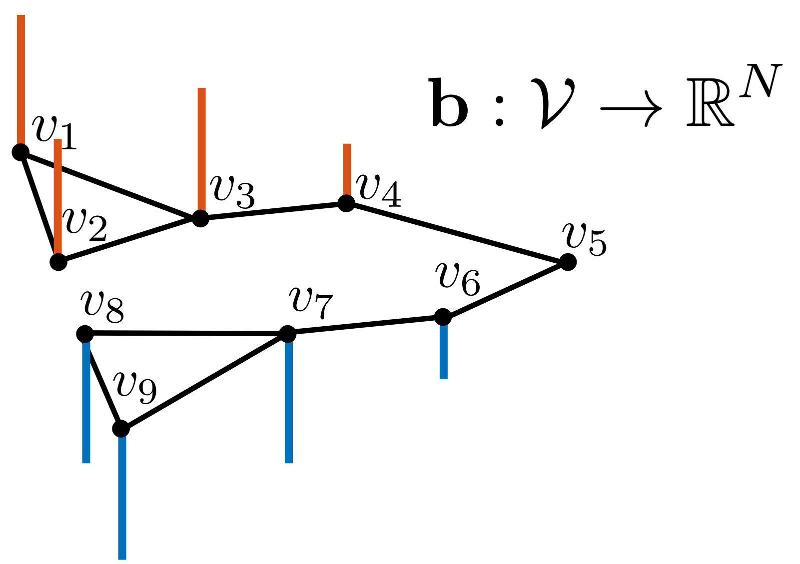

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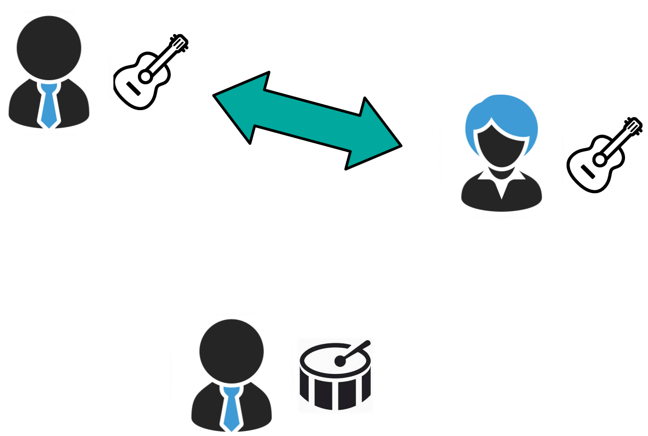

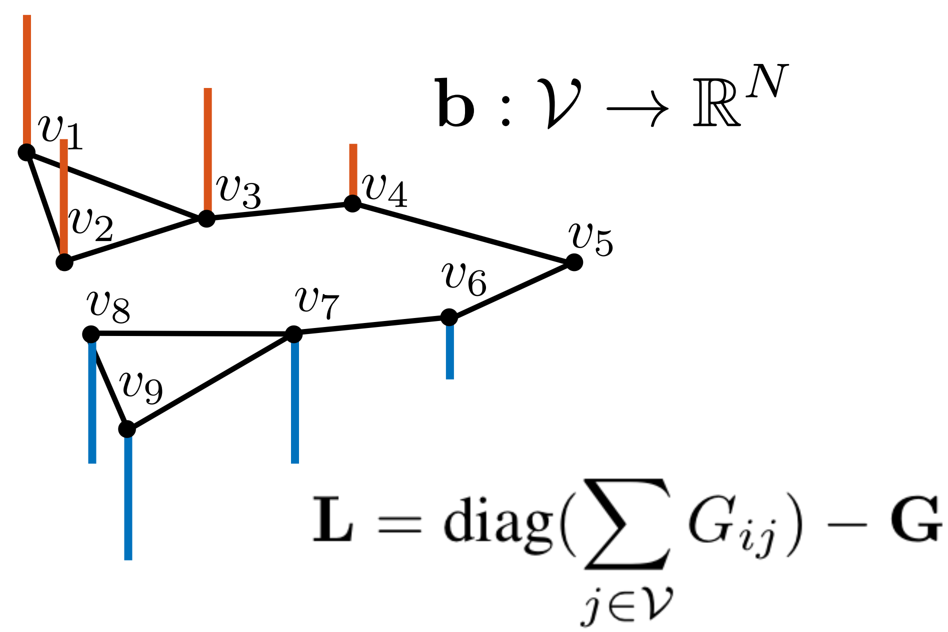

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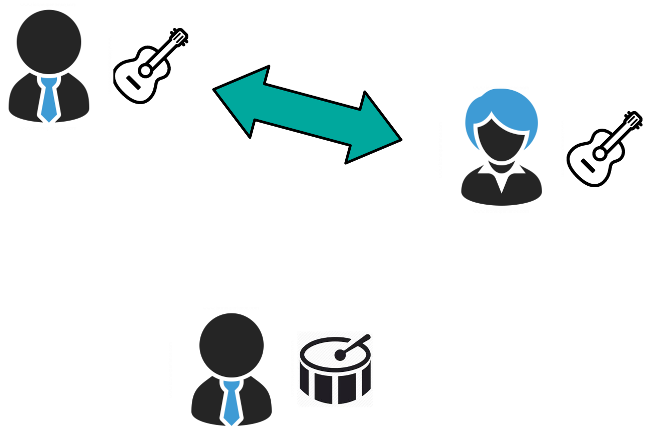

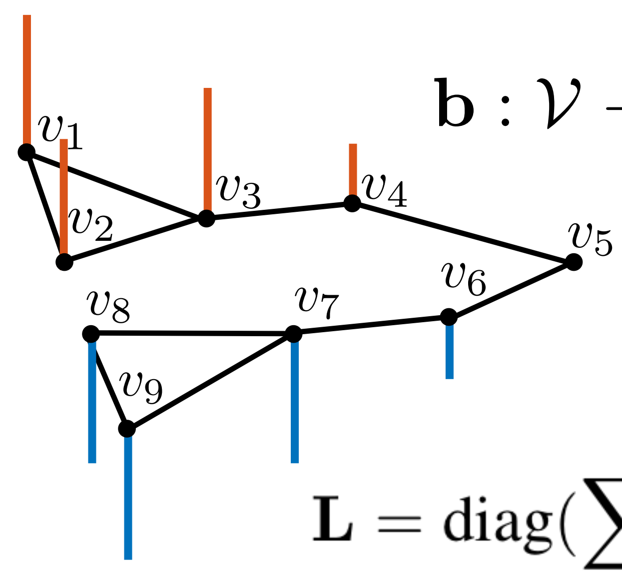

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\text{subject to} \quad & G_{ij} = G_{ji},\ G_{ij} \geq 0,\ G_{ii} = 0 \ \text{ for } \ \forall i,j \in \mathcal{V}, \\
& ||\mathbf{G}||_1 = N, \\
& \mathbf{L} = \text{diag}(\sum_{j\in\mathcal{V}} G_{ij}) - \mathbf{G},
\end{aligned}$$

Learning with homophilous marginal benefits

joint learning

$$\begin{aligned}
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\end{aligned}$$

- not jointly convex in G and B
- block coordinate descent
- convex in subproblems of solving for one while fixing the other

Remarks

$$\theta_2 \mathrm{tr}(\mathbf{B}^T \mathbf{L} \mathbf{B})$$

$$\begin{aligned} \underset{\mathbf{G},\mathbf{B}}{\text{minimize}} \quad & f(\mathbf{G}, \mathbf{B}) = ||(\mathbf{I} - \beta \mathbf{G})\mathbf{A} - \mathbf{B}||_F^2 + \theta_1 ||\mathbf{G}||_F^2 + \theta_2 ||\mathbf{B}||_F^2, \\ \text{subject to} \quad & G_{ij} = G_{ji},\ G_{ij} \geq 0,\ G_{ii} = 0 \ \text{ for } \ \forall i, j \in \mathcal{V}, \\ & ||\mathbf{G}||_1 = N, \end{aligned}$$

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- Strength of strategic interaction
  - $\rho(\ \mathbf{G})$ approaches 0: actions are independent from graph structure
  - $\rho(\ \mathbf{G})$ approaches 1: actions are proportional to eigenvector centrality (> 0)

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- Number of games
- Noisy observations

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Outline

- Setting
  - network games of linear-quadratic payoff
- Framework for learning network games
  - independent marginal benefits
  - homophilous marginal benefits
- Experimental results
  - synthetic data
  - real-world data
- Applications
- Discussions

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Experiments on synthetic data

- Random graphs
  - Erdős-Rényi (ER): edges created independently with certain probability
  - Watts-Strogatz (WS): regular graph followed by random rewiring
  - Barabási-Albert (BA): graph generated using preferential attachment
- Compute β so that $\rho(\ \mathbf{G}) \in (0,1)$
- Initialize marginal benefits for 50 games
  - independent: $\mathbf{b} \sim \mathcal{N}(\mathbf{0}, \mathbf{I} + \frac{1}{10}\mathbf{I})$
  - homophilous: $\mathbf{b} \sim \mathcal{N}(\mathbf{0}, \mathbf{L}^{\dagger} + \frac{1}{10}\mathbf{I})$
- Generate equilibrium actions $\mathbf{a} = (\mathbf{I} - \beta\mathbf{G})^{-1}\,\mathbf{b}$

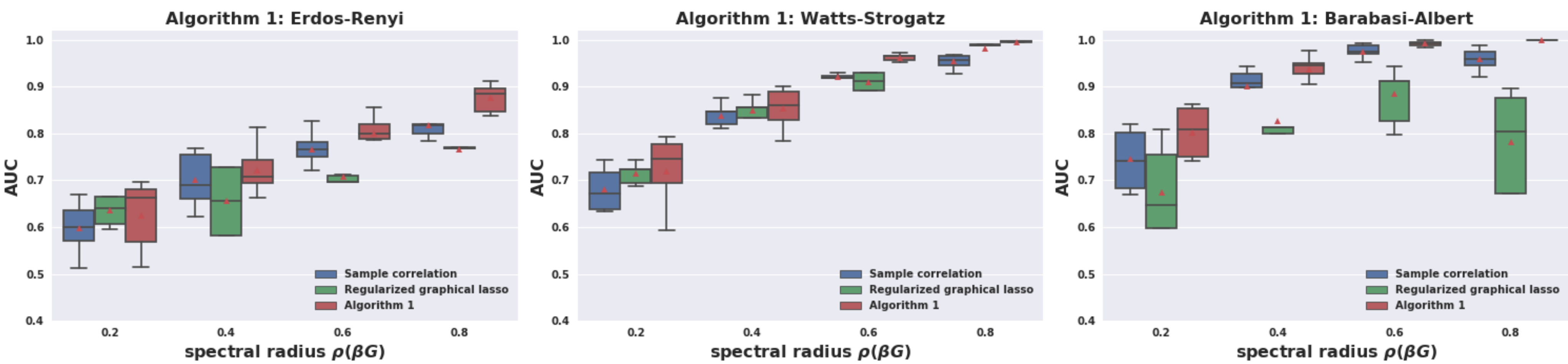
Setting

- Evaluate on area under the curve (AUC)
- Baselines (actions as input)
  - **sample correlation** as edge weights
  - graph learned by **regularized graphical Lasso** [Lake 2010]

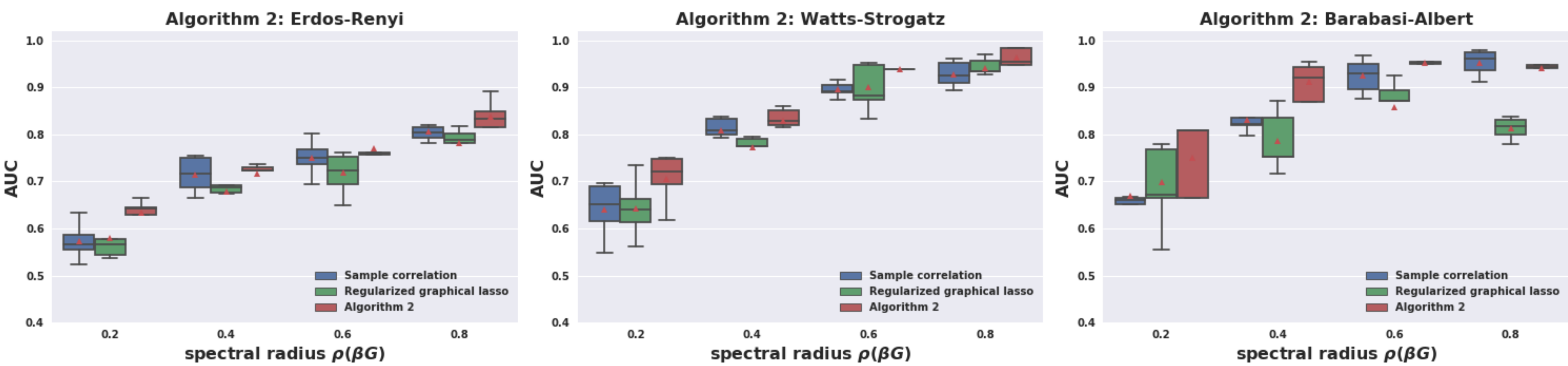
$$\begin{aligned} \underset{\boldsymbol{\Theta},\ \sigma^2}{\text{maximize}} \quad & \log\ \det\boldsymbol{\Theta} - \text{tr}(\frac{1}{M}\mathbf{X}\mathbf{X}^T\boldsymbol{\Theta}) - \rho||\boldsymbol{\Theta}||_1, \\ \text{subject to} \quad & \boldsymbol{\Theta} = \mathbf{L} + \frac{1}{\sigma^2}\mathbf{I},\ \mathbf{L} \in \mathcal{L}, \end{aligned}$$

Learning interaction network

independent marginal benefits

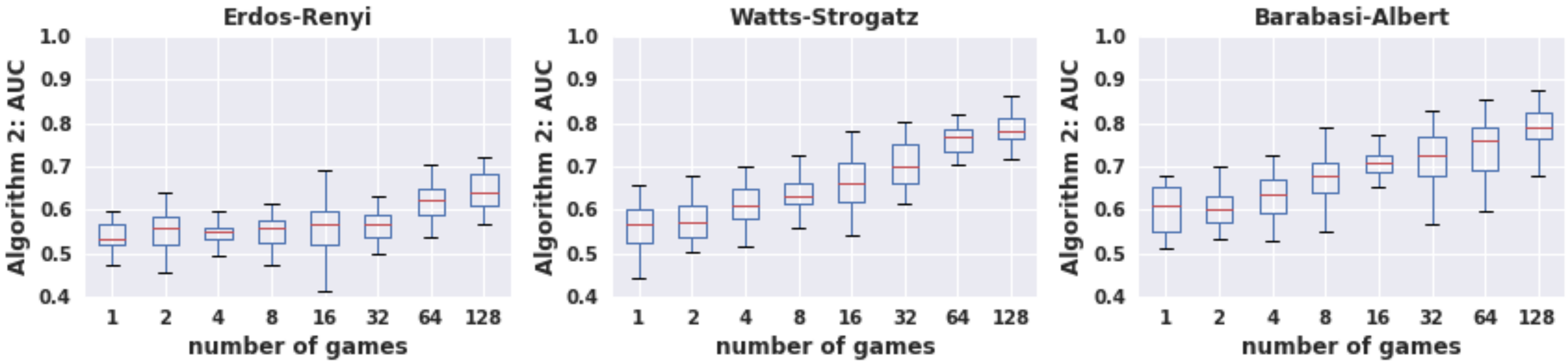


homophilous marginal benefits



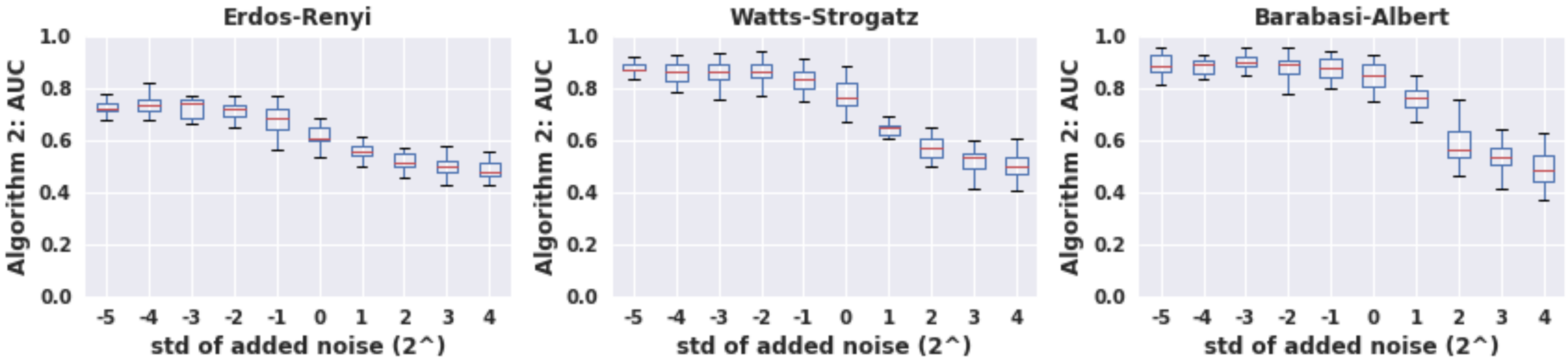
Performance vs. number of games

- Homophilous marginal benefits with $\rho(\mathbf{G}) = 0.6$



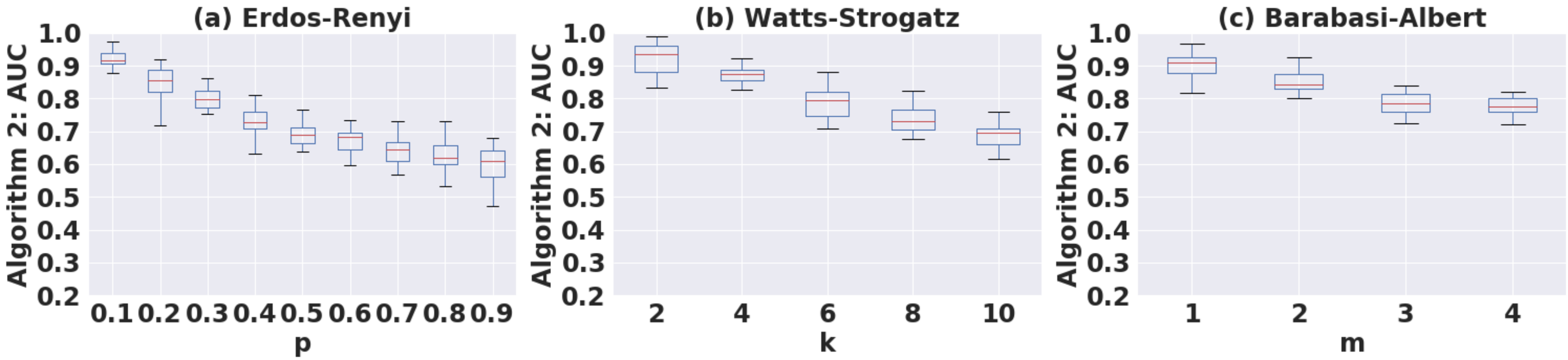
Performance vs. noise intensity

- Homophilous marginal benefits with $\rho(\quad \mathbf{G}) = 0.6$



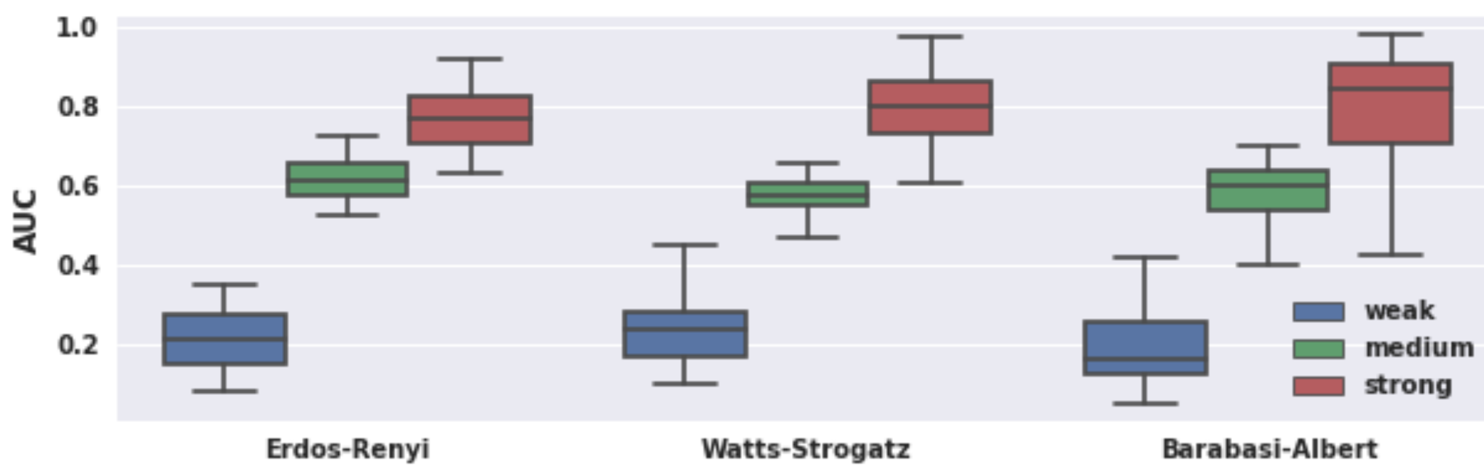
Performance vs. network structure

- Homophilous marginal benefits with $\rho(\mathbf{G}) = 0.6$
- Parameters in random graph models
  - ER: each node pair connected with probability p
  - WS: k-regular graph with rewiring probability p
  - BA: m nodes added at each graph generation step



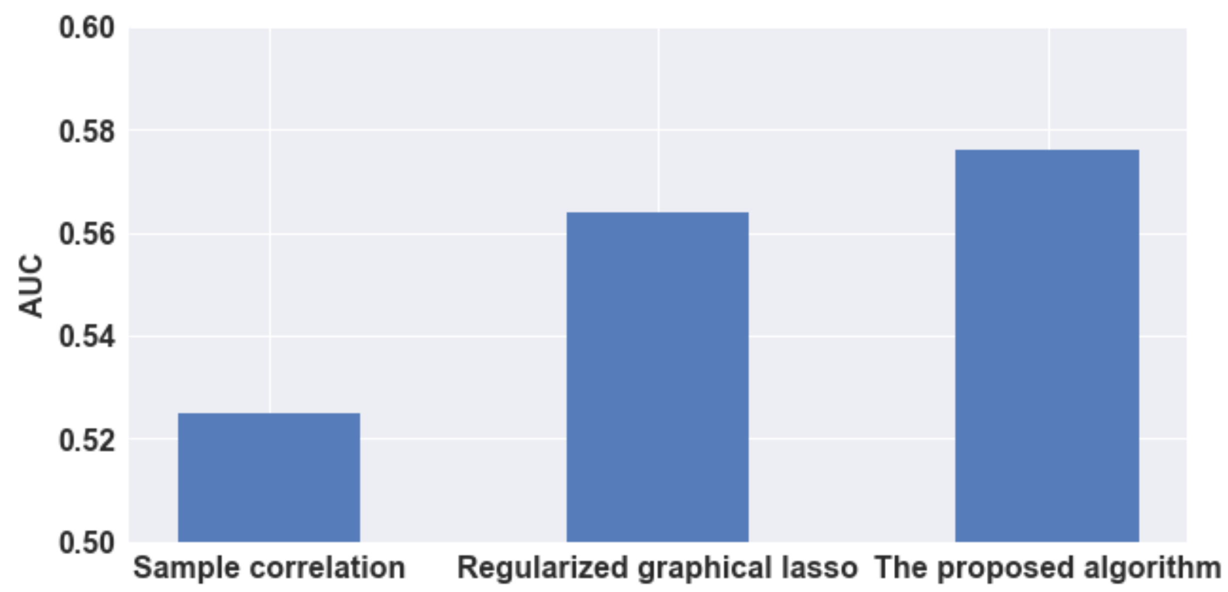
Performance vs. strength of homophily

- Homophilous marginal benefits with $\rho(\quad \mathbf{G}) = 0.6$
- Marginal benefits B as linear combinations of 1st-5th (strong homophily), 6th-10th (medium), 11th-15th (weak) eigenvectors of graph Laplacian



Experiments on real world data

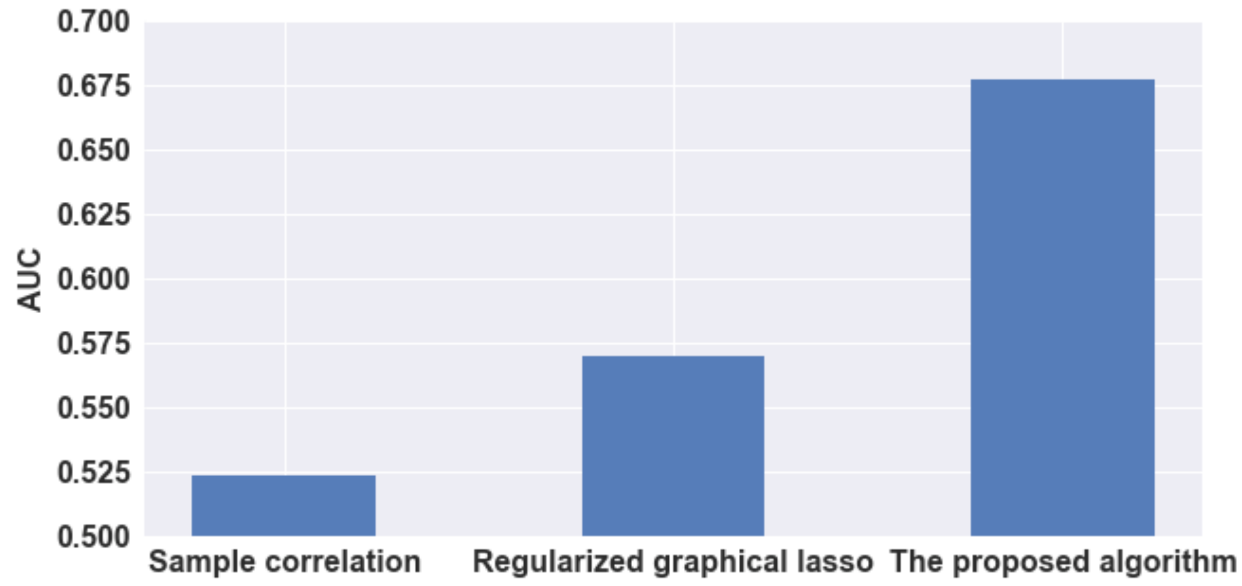
- Learning social network
  - 182 households in a village in rural India [Banerjee 2013]
  - actions in 31 games: number of facilities adopted by each household
  - complements: conformity to social norms (decisions made by neighbors)
  - compare with groundtruth self-reported friendship



Banerjee et al., "The diffusion of microfinance," Science, 2013.

Experiments on real world data

- Learning trade relationship
  - 235 countries
  - actions in 192 games: import/export of 96 products of countries in 2008
  - substitutes: more demand leads to less utility trading with high-demand countries (same applies to supply)
  - compare with groundtruth: total trades between each pair of countries in 2002



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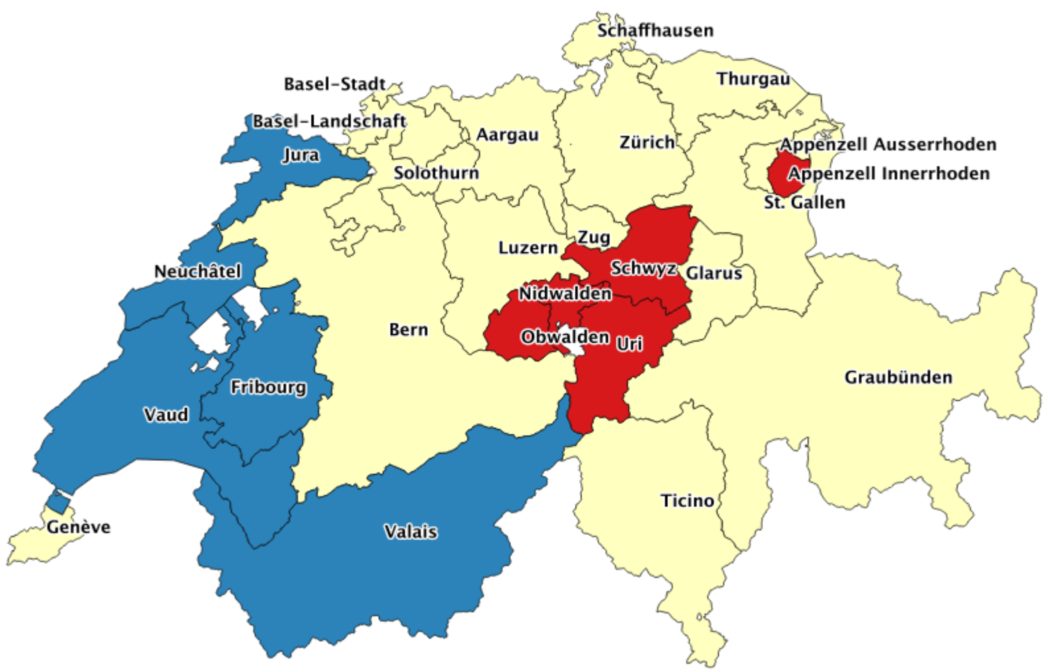
Application 1: community detection

- Setting
  - 26 cantons in Switzerland
  - actions in 37 games: percentage of supportive votes in referendums in 2008-2012
  - complements: political alliance

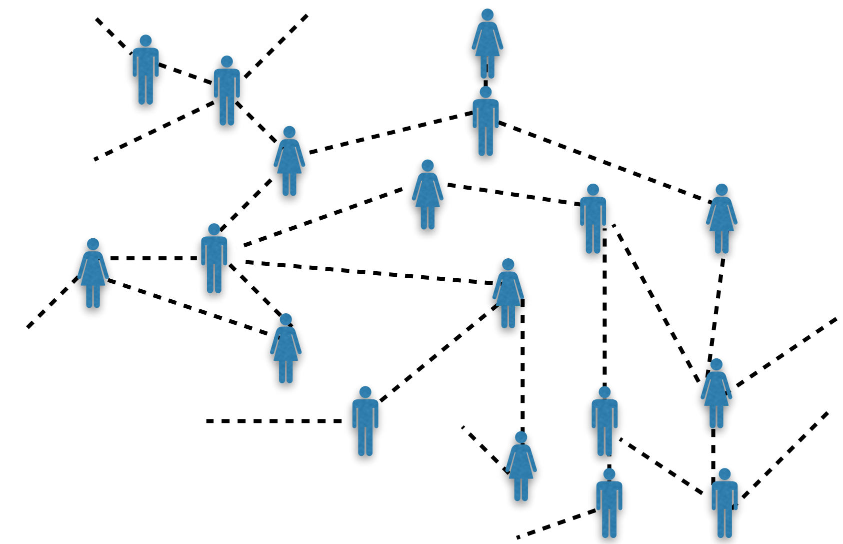
Application 1: community detection

- Setting
  - 26 cantons in Switzerland
  - actions in 37 games: percentage of supportive votes in referendums in 2008-2012
  - complements: political alliance
  - apply the spectral clustering algorithm [Ng 2001] to the inferred network



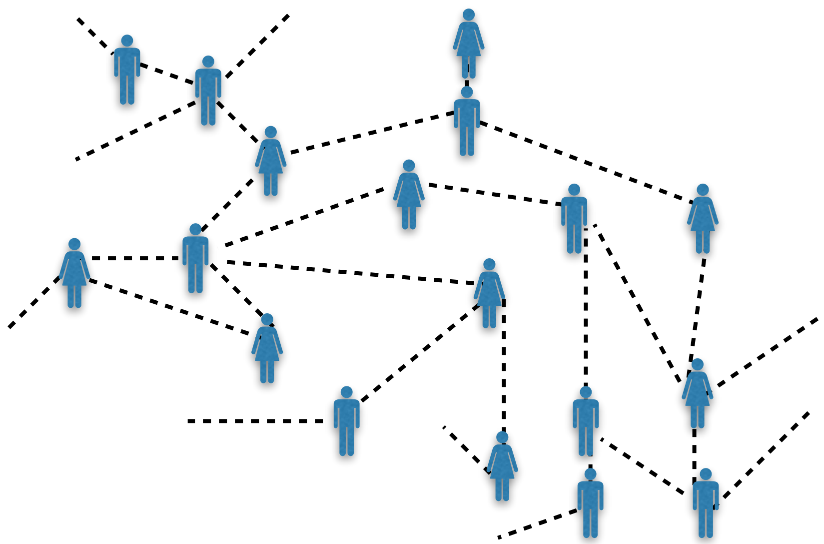
Application 2: vaccination

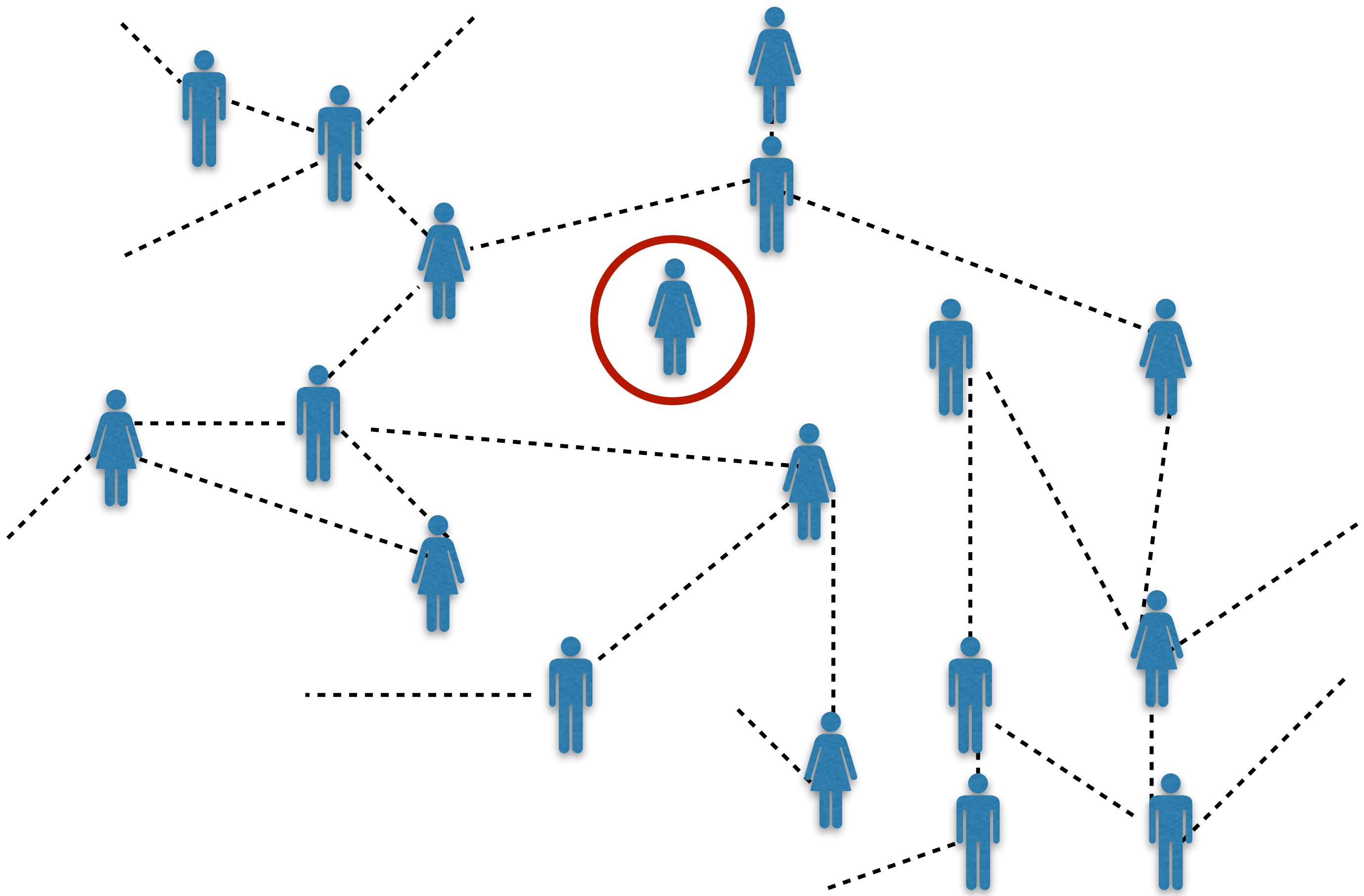
- Inoculate one individual in each village



Application 2: vaccination

- Inoculate one individual in each village
  - random pick





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- Inoculate one individual in each village
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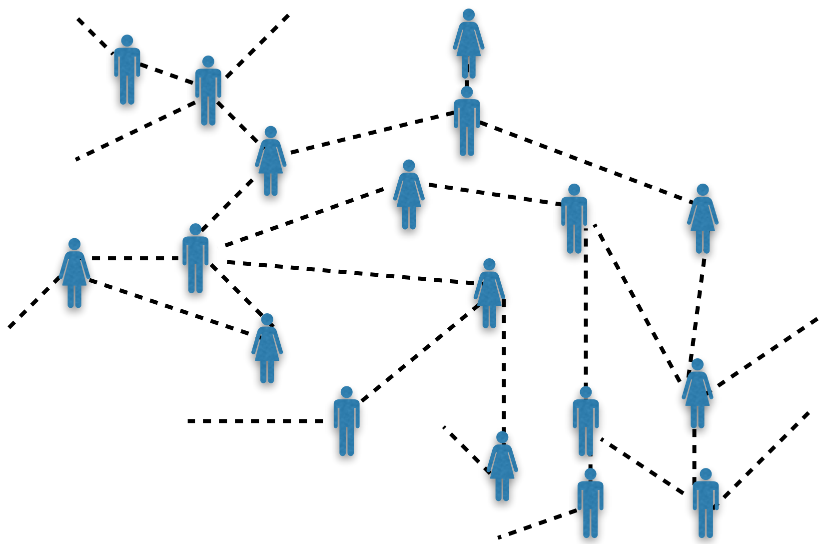
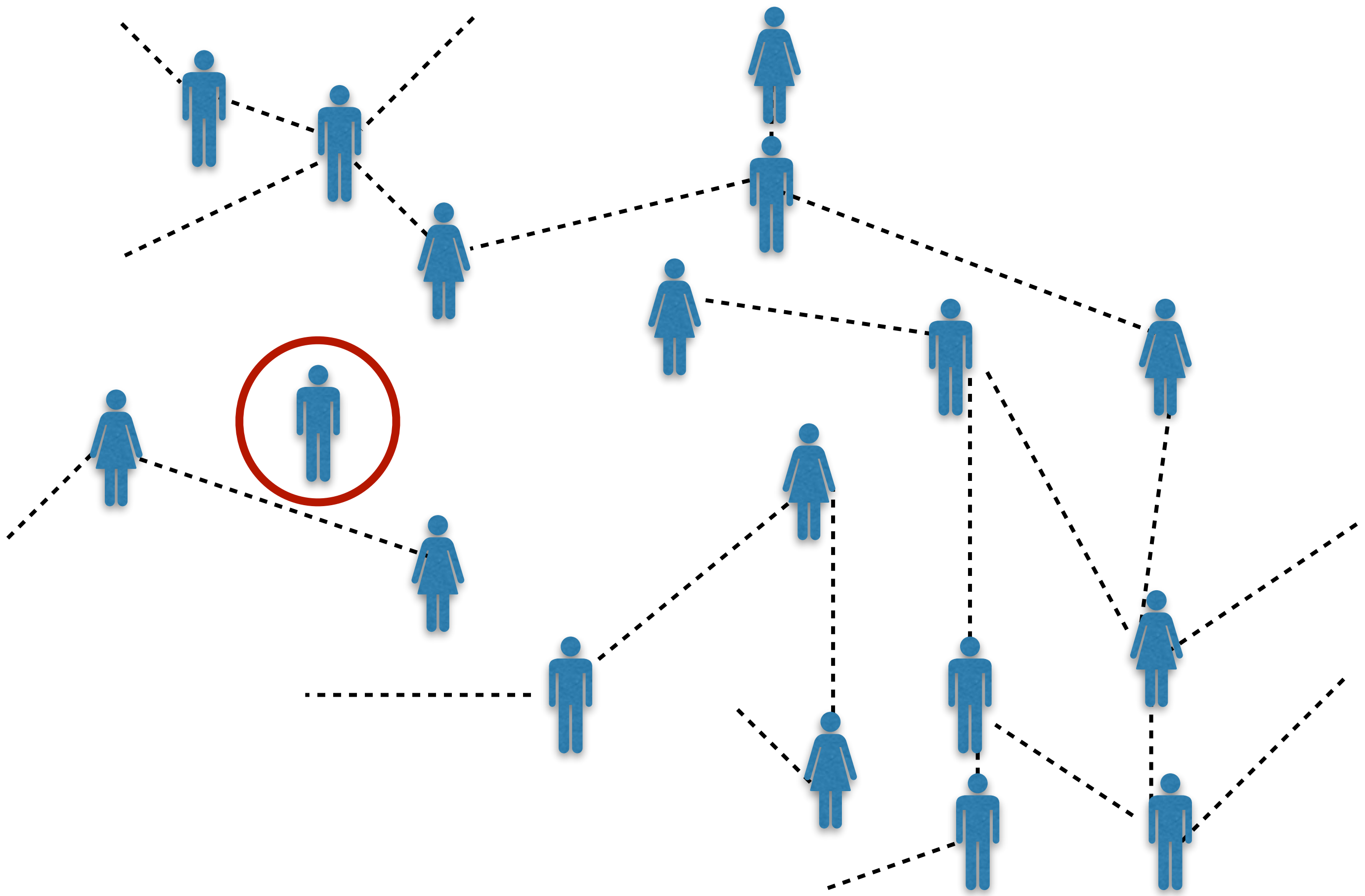
  - highest degree centrality





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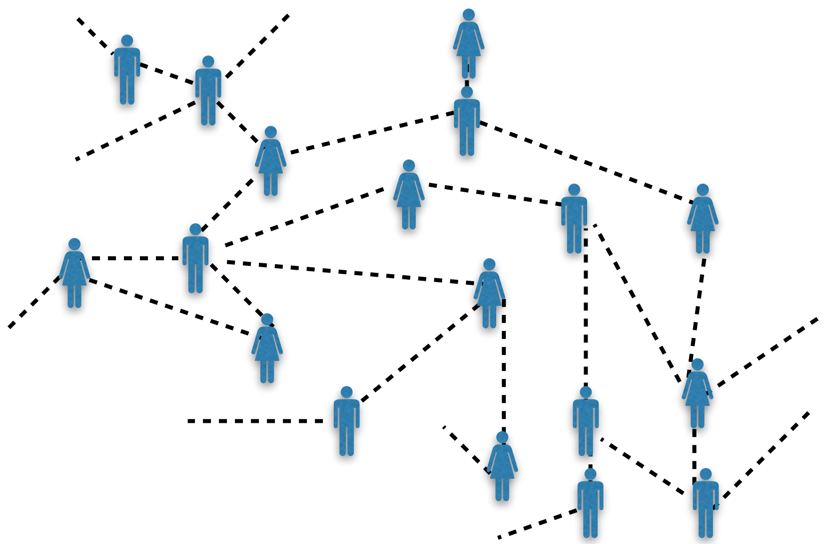
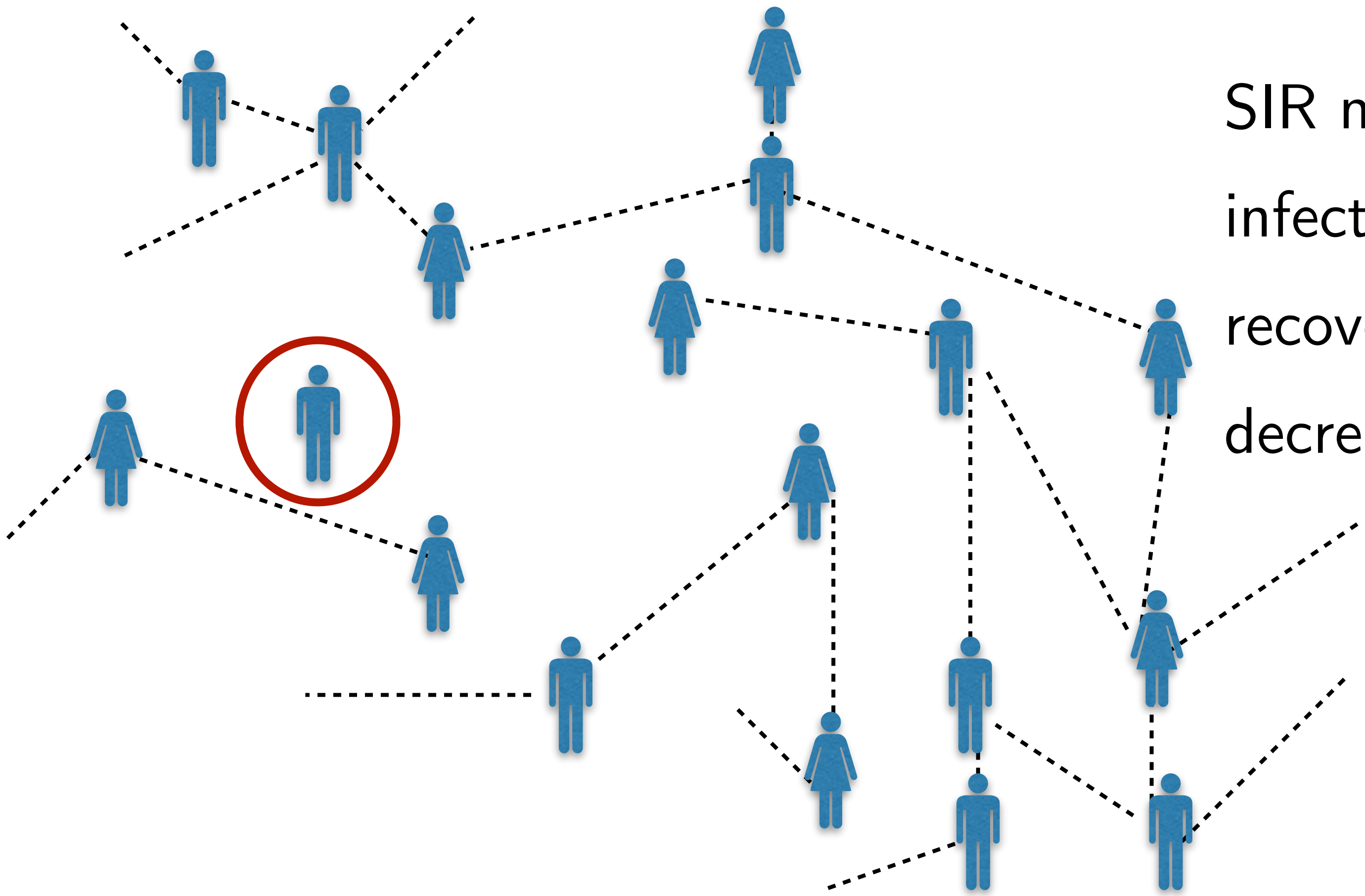




SIR model

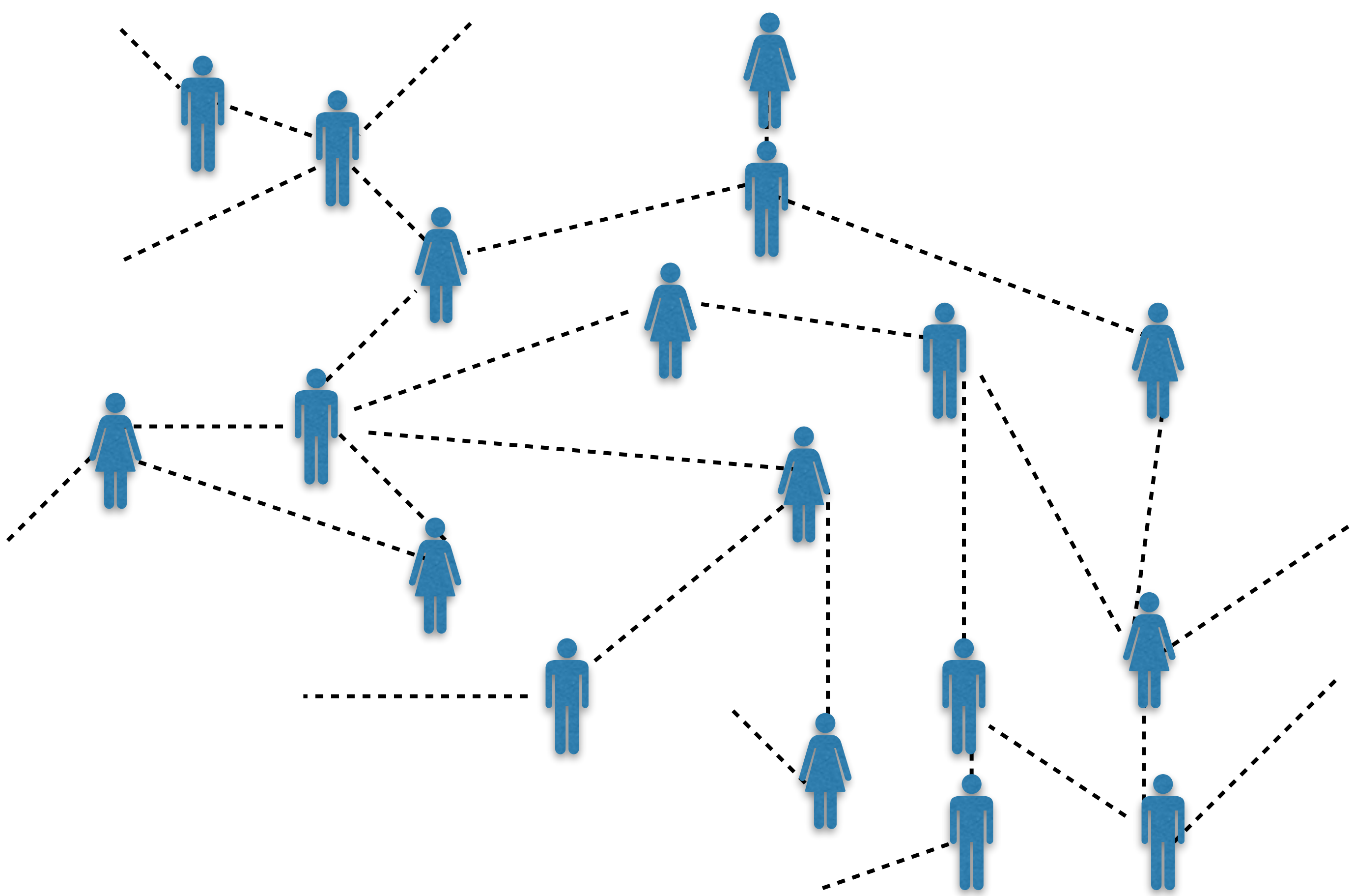
infection rate: 0.2

recover rate: 0.05

decreases in infection: 5.7%

Application 3: marketing campaign

- Send a product sample to one individual in each village



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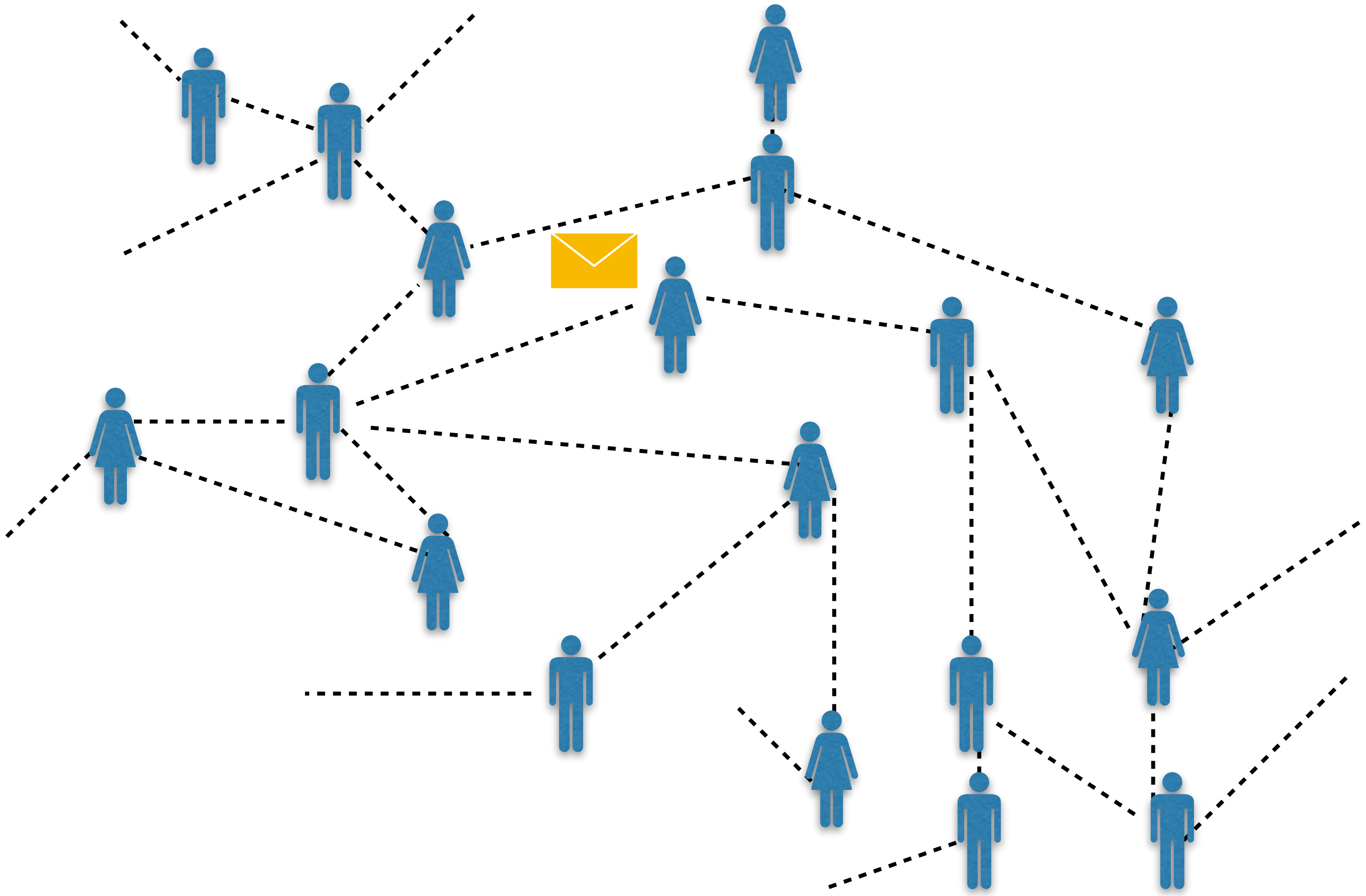

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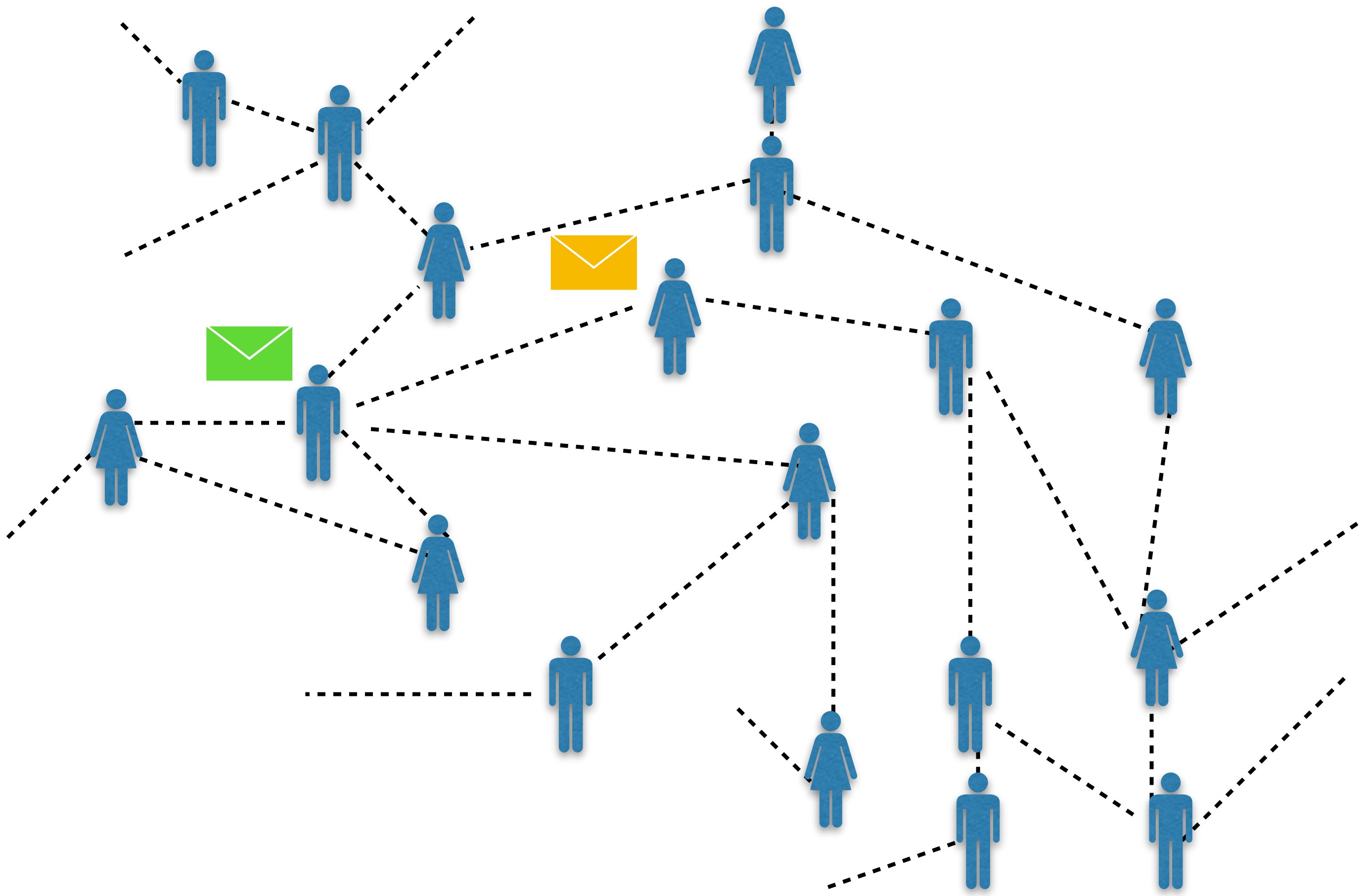

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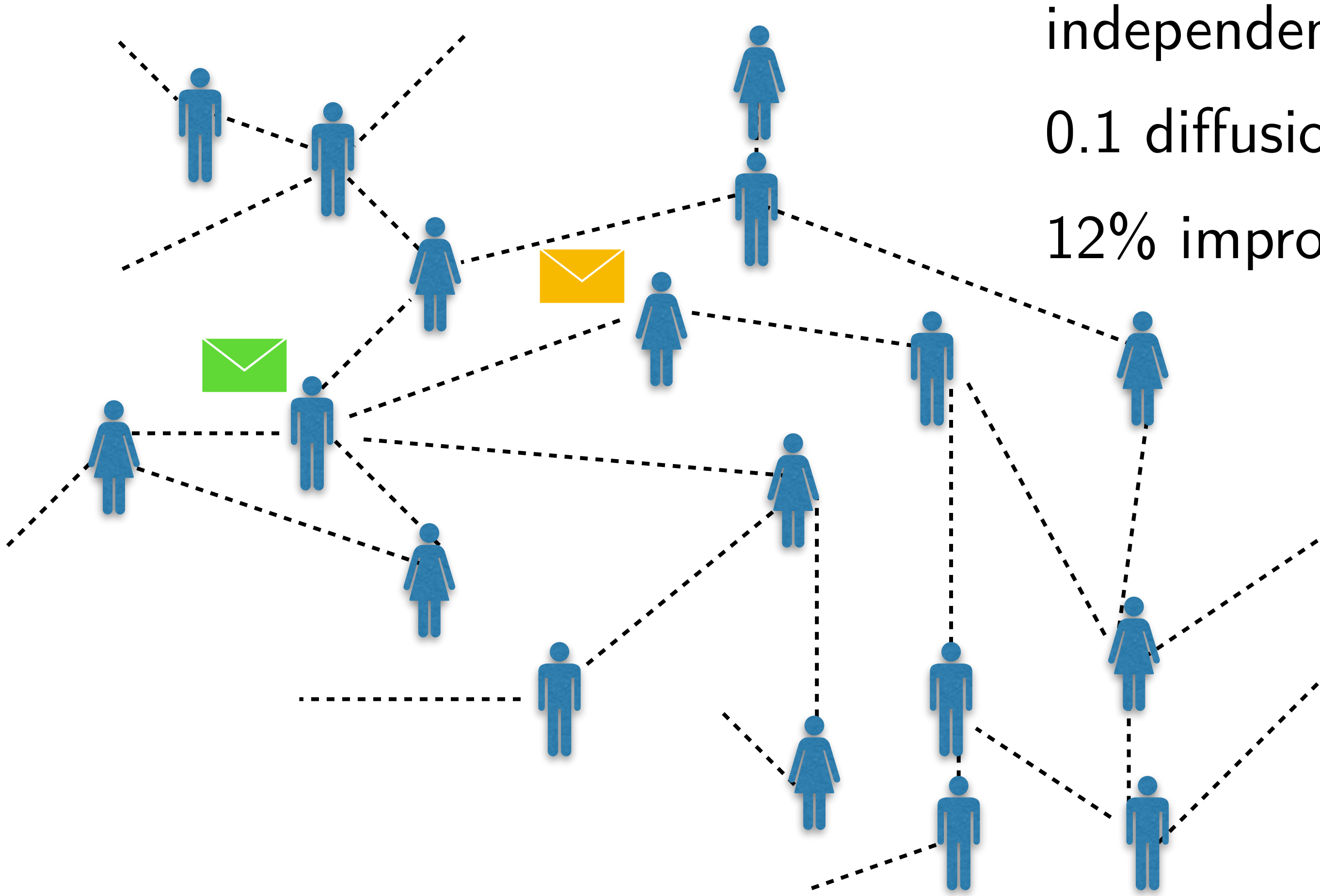

independent cascade model

0.1 diffusion rate

12% improvement in coverage

Application 4: design intervention

- Maximize aggregate payoffs of players [Galeotti 2017]
  - adjust marginal benefits to be proportional to eigenvector centrality
- Example:
  - maximize total payoffs from using a new technology
  - provide supplementary technologies

$$u_i = b_i a_i - \frac{1}{2} a_i^2 + \beta a_i \sum_{j \in \mathcal{V}} G_{ij} a_j$$

10k budget

1.76 times increase in payoffs

Discussion

- Contributions
  - a **novel formulation and framework** to infer underlying **strategic relationship** and **marginal payoffs** from decision-makings
  - shown to be **effective** in synthetic and real-world settings
  - a building block for a wide range of **applications**
- Open issues & future directions
  - theoretical understanding, e.g., recovery guarantee
  - more general payoff functions
  - partial/incomplete observations
  - dynamic interaction networks
  - real-world interventions